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Theoretical evidence for a tachyonic ghost state contribution to the gluon propagator
in high energy, forward quark-quark 'scattering'*

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Abstract

Implications stemming from the inclusion of non-perturbative, confinining effects, as contained in the Stochastic Vacuum Model of Dosch and Simonov, are considered in the context of a, hypothetical, quark-quark 'scattering process' in the Regge kinematical region. In a computation wherein the non-perturbative input enters as a correction to established perturbative results, a careful treatment of infrared divergencies is shown to imply the presence of an effective propagator associated with the existence of a linear term in the static potential. An equivalent statement is to say that the modified gluonic propagator receives contribution from a tachyonic ghost state, an occurence which is fully consistent with earlier such suggestions made in the context of low energy QCD phenomenology.

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1. Introduction

From the theoretical point of view, forward scattering at very high energies (Regge kinematics) presents a situation where one can readily apply eikonal approximation techniques [1]. In this context, such processes provide grounds for exploring long distance properties of the underlying fundamental theory for the implicated interaction. For the particular case of QCD, long distance behavior constitutes a fundamental issue whose exploration is not only relevant to high energy processes but also to low energy phenomenology.

In the present paper, we revisit the (idealized) problem of quark-quark 'scattering' in the Regge limit, which has been extensively studied within the framework of pQCD [1-5], with the aim to extend the aformentioned analyses in a direction which takes into account confining aspects of the theory. Specifically, we shall rely on the premises of the Stochastic Vacuum Model (SVM), pioneered by Dosch and Simonov [6] for the explicit purpose of accomodating the confinement property of QCD. A similar approach has been pursued by Nachtmann [7] by employing a different methodology from the one we shall adopt in this work. Different will also be the the scope of the present analysis.

The construction of the SVM is motivated by the intention to incorporate established observations/results regarding the structure of the QCD vacuum [8] into a well defined theoretical framework. In particular, it summarizes all that is known and/or surmized about its properties through a set of three axioms, which are expressed in terms of field strength, as opposed to field potential, correlators. The underlying stochasticity assumption for the vacuum state facilitates the application of the cumulant expansion [9], which describes the factorization rules for higher order gluon field strength correlators in terms of two-point ones. One of the first results arrived at through the SVM is the deduction of the area law for the static Wilson loop, i.e. confinement. Specific applications of the SVM scheme, including comparisons with lattice results can be found, e.g., in Ref [10].

A concrete, as well as practical, way to apply the SVM scheme to specific situations has been suggested by Simonov [11]. The idea is to use the background gauge fixing method [12] and assign the background gauge fields with the task of becoming the agents of the non-perturbative dynamics. Specifically, one employs the gauge potential splitting $A^a_\mu = \alpha^a_\mu + B^a_\mu$ with the α^a_μ being associated with the usual perturbative field modes. The B^a_μ, on the other hand, enter as *dynamical* fields, assigned with the task of carrying the non-perturbative physics through field strength correlators which adhere to the cumulant expansion rules.

Following our previous work of Ref [5] we find it convenient to employ the FFS-worldline casting of QCD, originally pioneered by Fock [13], Feynman [14] and Schwinger [15] and most recently revived in path integral versions, see Refs [16-18]. The reason for such a choice is that the eikonal approximation acquires a straightforward realization in this scheme, since, in the perturbative context at least, it can be readily implemented by restricting one's considerations to straight worldline paths. As it will turn out, the inclusion of input from the SVM will produce a kind of deformation of the eikonal paths, with low energy consequences, which represent, subleading, perturbative-nonperturbative interference effects. The conditions which justify the relevant computation will be made explicit in the text. Suffice it to say, at this point, that it was explicitly demonstrated in Ref [19] that the FFS-worldline formulation of QCD, in combination with the background gauge field splitting, is ideally suited for providing a framework within which one can directly and efficiently apply

the premises of the SVM.

The main result of this paper is the following: Once (subleading) contributions incorporating nonperturbative, as induced by the SVM, corrections to the perturbative expressions are taken into account, then a consistent analysis of the infrared issues entering the quark-quark high energy forward 'scattering' process reveals that the gluonic propagator exhibits a behavior which can be interpreted in terms of the presence of a 'tachyonic mass pole'. Arguments in favor of such an occurence, with important phenomenological as well as theoretical implications, have been promoted, from different perspectives, in several papers [20-22].

The organization of the paper is as follows. In the next section we present the basic formulas related to the 'amplitude' for the high energy 'quark-quark scattering process' in the forward direction and display their FFS-worldline form. Section 3 focuses its attention on infrared issues associated with the pertubative-nonperturbative interference effects under consideration in this study. The resulting expression for the amplitude is shown to be equivalent to the introduction of an effective propagator, which is associated with the presence of a linear term in the static potential. Section 4 extends the implications of the aformentioned result to issues related to renormalization: Modified running coupling constant and summation of leading logarithms via the Callan-Symanzyk equation. Finally, the technical manipulations leading to the main result of section 3 are displayed in the appendix.

2. Preliminary considerations

Consider an idealized quark-quark 'scattering' process in the Regge limit, defined by $s/m^2 \to \infty$, $s \gg t (= -q^2 = \vec{q}_\perp{}^2)$. The amplitude is given by

$$T_{ii'\,jj'}(s/m^2, q_\perp^2/\lambda^2) = \int d^2b\, e^{-i\vec{q}\cdot\vec{b}} E_{ii'\,jj'}(s/m^2, 1/b^2\lambda^2), \tag{1}$$

where λ is an 'infrared' scale, b is the impact distance in the transverse plane to the direction of the colliding quarks (assumed to be travelling along the x_3 axis) while $E_{ii'\,jj'}$, which incorporates the dynamics of the process, is specified, in Euclidean space-time, by [5]

$$E_{ii'\,jj'} = \left\langle \mathrm{P}\exp\left[ig\int_{-\infty}^{\infty} d\tau\, v_1\cdot A(v_1\tau)\right]_{ii'} \mathrm{P}\exp\left[ig\int_{-\infty}^{\infty} d\tau\, v_2\cdot A(v_2\tau+b)\right]_{jj'}\right\rangle_A^{\mathrm{conn}}. \tag{2}$$

The v_1 and v_2 are constant four velocities characterizing the respective eikonal lines of the quarks participating in the 'scattering process' and 'conn' stands for 'connected'. In Minkowski space (and in light cone coordinates) one has $v_1 \simeq (v_1^+, 0, \vec{0}_\perp)$, $v_2 \simeq (0, v_2^-, \vec{0}_\perp)$, $b \simeq (0, 0, \vec{b}_\perp)$ with $v_1^+ \simeq v_2^- \simeq \frac{1}{\sqrt{2}}\sqrt{s}/m$.

Employing the gauge field splitting $A_\mu^a = \alpha_\mu^a + B_\mu^a$, the expectation values with respect to field configurations acquire the form $<\cdots>_A = <\cdots>_{\alpha,B}$. Expanding in terms of powers of the perturbative field components, one obtains

$$E_{ii'\,jj'} = \left\langle \mathrm{P}\exp\left[ig\int_{-\infty}^{\infty} d\tau\, v_1\cdot B(v_1\tau)\right]_{ii'} \mathrm{P}\exp\left[ig\int_{-\infty}^{\infty} d\tau\, v_2\cdot B(v_2\tau+b)\right]_{jj'}\right\rangle_B^{\mathrm{conn}}$$
$$-g^2\int_{-\infty}^{\infty} ds_2 \int_{-\infty}^{\infty} ds_1 \Big\langle \mathrm{P}\exp\left[ig\int_{s_1}^{\infty} d\tau\, v_1\cdot B(v_1\tau)\right]_{ik} \mathrm{P}\exp\left[ig\int_{-\infty}^{s_1} d\tau\, v_1\cdot B(v_1\tau)\right]_{li'}$$

$$\times \mathrm{P}\exp\left[ig\int_{s_2}^{\infty} d\tau\, v_2\cdot B(v_2\tau+b)\right]_{jm} \mathrm{P}\exp\left[ig\int_{-\infty}^{s_2} d\tau\, v_2\cdot B(v_2\tau+b)\right]_{nj'}$$

$$\times t^a_{kl} t^b_{mn} v_{2\mu} v_{1\nu} iG^{ba}_{\mu\nu}(v_2 s_2+b,\, v_1 s_1)\Big\rangle_B + \mathcal{O}(g^4 < \alpha^4 >). \tag{3}$$

Our objective in this paper is to study the behavior of the amplitude as $|b| \to 0$. Accordingly, contributions attributed exclusively to the non-perturbative, background terms will be ignored given that, by definition, they are finite in this limit. This narrows the expression of computational interest to the following one

$$\begin{aligned} E_{ii'\,jj'} &= -g^2 \frac{N_C}{N_C^2-1} t^a_{ii'} t^a_{jj'} \int_{-\infty}^{\infty} ds_2 \int_{-\infty}^{\infty} ds_1 v_{2\mu} v_{1\nu} \left\langle \frac{1}{N_C} Tr_A iG_{\mu\nu}(v_2 s_2+b,\, v_1 s_1)\right\rangle_B \\ &\quad + \mathcal{O}(g^4 < \alpha^4 >) + \mathcal{O}(b^2) \end{aligned} \tag{4}$$

The propagator $iG^{ba}_{\mu\nu}(v_2 s_2+b,\, v_1 s_1) \equiv < \alpha^b_\mu(v_2 s_2+b)\alpha^a_\nu(v_1 s_1) >$ acquires the following worldline expression

$$iG^{ba}_{\mu\nu}(v_2 s_2+b,\, v_1 s_1) = \int_0^\infty dT \int\limits_{\substack{x(0)=v_1 s_1 \\ x(T)=v_2 s_2+b}} \mathcal{D}x(t) e^{-\frac{1}{4}\int_0^T dt \dot{x}^2(t)} \mathrm{P}\left(e^{g\int_0^T dt J\cdot F + ig\int_0^T dt \dot{x}\cdot B}\right)^{ba}_{\mu\nu}. \tag{5}$$

In the above formula $(J\cdot F)_{\mu\nu} = J^{\alpha\beta}_{\mu\nu} F_{\alpha\beta}$, with $J^{\alpha\beta}_{\mu\nu}$ the generators for the spin-1 representation of the Lorentz group[1]. It should also be noted that we have employed the notation $B^{ba}_\mu = B^c_\mu (t^c_G)^{ab} = -B^c_\mu f^{abc}$.

To close this section let us briefly comment on gauge symmetry related issues. Given the 'idealized' process under consideration our handling of gauge invariance will be to let the two worldlines of the 'colliding' quarks to extend to infinity in both directions and impose the boudary conditions $A_\mu[x(t)] \to 0$ as $t_{\mathrm{Eucl}} \to \pm\infty$. In this way, the overall worldline configuration introduces a Wilson loop in the path integrals, given that the end-points of the two trajectories can now be joined at $+\infty$ and at $-\infty$. These, of course, do not correspond to boundary conditions for a scattering process *per se*, however our only objective in this work is to extract long distance implications based on the exchanges taking place in the immediate vicinity of the points of closest approach between the two worldlines. The study of realistic situations involving the scattering of physically observable particle entities in the Regge limit, using the presently adopted methodology, is under current consideration and the relevant analysis will be presented in the near future. Finally, concerning the issue of gauge fixing for the B field sector, our choice is prompted by the intention to rely on field strength correlators for describing nonperturbative dynamics [11]. It, accordingly, becomes convenient to work in the Fock-Schwinger (F-S) gauge [23, 24]. The latter is specified by

$$B^a_\mu(x) = -\int_{x_0}^x du_\nu (\partial_\mu u_\rho) F^a_{\mu\nu}(u) = -(x-x_0)_\nu \int_0^1 d\alpha\, \alpha F_{\mu\nu}(x_0+\alpha(x-x_0)). \tag{6}$$

[1]Its incorporation into the worldline form of the propagator serves to signify the spin of the accomodated modes.

Of course, the arbitrary point x_0 should not enter any gauge invariant expression.

3. Infrared issues associated with the propagation of gluonic modes in a confining environment

Consider the quantity defined by

$$I(l) \equiv \frac{1}{N_C^2-1} v_{2\mu} v_{1\nu} \langle i Tr_A G_{\mu\nu}(l)\rangle_B = \frac{N_C}{N_C^2-1} v_{2\mu} v_{1\nu}$$
$$\times \int_0^\infty dT \int\limits_{\substack{x(0)=0\\x(T)=l}} \mathcal{D}x(t) e^{-\frac{1}{4}\int_0^T dt \dot{x}^2(t)} \left\langle \frac{1}{N_C} Tr_A \mathrm{P} \exp\left(g\int_0^T dt J\cdot F + ig\int_0^T dt \dot{x}\cdot B\right)_{\mu\nu}\right\rangle_B \quad (7)$$

where we have introduced $l_\mu \equiv v_{2\mu}s_2 - v_{1\mu}s_1 - b_\mu$. It describes the propagation of the perturbative gluon modes in the presence of the background gauge field modes B_μ^a and, in this sense, it is expected to incorporate confinement effects associated with the SVM.

Generally speaking, one would expect that in a study of a physically relevant process with the full (and proper) inclusion of non-perturbative effects, no need would arise for the introduction of an infrared cutoff to regulate the various expressions entering the computation at long distances. An infrared scale should, in other words, naturallly arise suppressing contributions from the very large distances ($|l| \to \infty$). Given that the object of the present study is to investigate perturbative/non-perturbative interference, long distance effects in the (nonphysical) process of quark-quark high energy 'collision' in the forward direction, it becomes necessary to regulate infrared divergences associated with the upper limit of the T-integral. Our choice of introducing the infrared cutoff is via the relacement $\int_0^\infty(\cdots)dT \to \int_0^\infty dT e^{-T\lambda^2}(\cdots)$. On a simple dimensional basis and given the length scales entering the problem, one could make the association $\lambda \propto \sigma|l|$.

Upon expanding the exponential one obtains

$$I(l) = \frac{v_1\cdot v_2}{4\pi^2|l|^2} - \frac{2N_C^2}{N_C^2-1} v_{2\mu} v_{1\nu} \int_0^\infty dT\, e^{-T\lambda^2} \int_0^T dt_2 \int_0^T dt_1\, \theta(t_2-t_1)$$
$$\times \int\limits_{\substack{x(0)=0\\x(T)=l}} \mathcal{D}x(t) e^{-\frac{1}{4}\int_0^T dt\dot{x}^2(t)} \Big\{ \delta_{\mu\nu} \frac{1}{N_C} Tr_F \langle g\dot{x}(t_2)\cdot B(x(t_2)) g\dot{x}(t_1)\cdot B(x(t_1))\rangle_B$$
$$+ \frac{2}{N_C} Tr_F \langle gF_{\mu\rho}(x(t_2) gF^c_{\rho\nu}(x(t_1)\rangle_B \Big\} + \mathcal{O}(\langle g^4F^4\rangle_B) \quad (8)$$

Observe, now, that in the F-S gauge, the following relation holds

$$Tr_F\langle gB_{\mu_2}(x(t_2)) gB_{\mu_1}(x(t_1))\rangle_B = (x_2-x_0)_{\nu_2}(x_1-x_0)_{\nu_1} \int_0^1 d\alpha_2\alpha_2 \int_0^1 d\alpha_1\alpha_1$$
$$\times Tr_F\langle gF_{\mu_2\nu_2}(x_0+\alpha_2(x_2-x_0)) gF_{\mu_1\nu_1}(x_0+\alpha_1(x_1-x_0))\rangle_B, \quad (9)$$

which brings into play the field strength correlator.

Setting $u_i = x_0 + \alpha_i x(t)$, $i=1,2$ one writes

$$\frac{1}{2N_C}\langle gF^c_{\mu_2\nu_2}(u_2) gF^c_{\mu_1\nu_1}(u_1)\rangle_B = \frac{1}{N_C} Tr_F\langle \phi(x_0,u_2) gF_{\mu_2\nu_2}(u_2)$$
$$\times \phi(u_2,x_0)\phi(x_0,u_1) gF_{\mu_1\nu_1}(u_1)\phi(u_1,x_0)\rangle_B \equiv \Delta^{(2)}_{\mu_2\nu_2,\mu_1\nu_1}(u_2-u_1), \quad (10)$$

where $\phi(x_0, u_i) = \mathrm{P}exp\left(ig\int_{u_i}^{x_0} dv \cdot B(v)\right)$ and is unity in the F-S gauge. Its insertion serves to underline the gauge invariance of the field strength correlator.

With the above in place and upon making the redefinition $t_i \to Tt_i$, $i = 1, 2$, one determines

$$I(l) = \frac{v_1 \cdot v_2}{4\pi^2|l|^2} - \frac{2N_C^2}{N_C^2 - 1} v_1 \cdot v_2 \int_0^1 d\alpha_2 \alpha_2 \int_0^1 d\alpha_1 \alpha_1 \int_0^1 dt_2 \int_0^1 dt_1\, \theta(t_2 - t_1)$$
$$\times \int_0^\infty dT T^2 e^{-T\lambda^2} \int\limits_{\substack{x(0)=0 \\ x(T)=l}} \mathcal{D}x(t) e^{-\frac{1}{4}\int_0^1 dt \dot{x}^2(t)} \left\{ 16 \frac{v_{2\mu} v_{1\nu}}{v_2 \cdot v_1} \Delta^{(2)}_{\mu\rho,\rho\nu}(x(t_2) - x(t_1)) \right.$$
$$\left. + \frac{1}{T^2} \dot{x}_{\mu_2}(t_2) x_{\nu_2}(t_2) \dot{x}_{\mu_1}(t_1) x_{\nu_1} x_(t_1) \Delta^{(2)}_{\mu_2\nu_2,\mu_1\nu_1}[\alpha_2 x(t_2) - \alpha_1 x(t_1)] \right\} + \mathcal{O}(\langle g^4 F^4 \rangle_B). \quad (11)$$

On a kinematic basis, the correlator can be represented as follows [6,10]

$$\Delta^{(2)}_{\mu_2\nu_2,\mu_1\nu_1}(z_2 - z_1) = (\delta_{\mu_2\mu_1}\delta_{\nu_2\nu_1} - \delta_{\mu_2\nu_1}\delta_{\nu_2\mu_1}) D(z^2)$$
$$+ \frac{1}{2}\frac{\partial}{\partial z_{\mu_1}}\left[(z_{\mu_2}\delta_{\nu_2\nu_1} - z_{\nu_2}\delta_{\mu_2\nu_1}) D_1(z^2)\right] + \frac{\partial}{\partial z_{\nu_1}}\left[(z_{\nu_2}\delta_{\mu_2\mu_1} - z_{\mu_2}\delta_{\nu_2\mu_1}) D_1(z^2)\right]. \quad (12)$$

One notices [10,11] that the first term enters as a distinct feature of the non-abelian nature of the gauge symmetry (it is not present, e.g., in QED). According to the premises of the SVM it is associated with the (QCD) string tension σ by [10,11]

$$\int_0^\infty dz^2 D(z^2) = \frac{1}{\pi}\int d^2 z D(z^2) \equiv \frac{2}{\pi}\sigma. \quad (13)$$

Now, the central objective the present analysis is to determine first order contributions to the amplitude coming, via the SVM, from the non-perurbative/confining sector of QCD. The corresponding, lowest order correction is expected, on dimensional grounds, to be of the string tension σ. This imlplies, as already pointed out by Simonov [25], that we shall set aside the D_1 term entering the kinematical analysis of the correlator, according to Eq (12), which cannot furnish contributing terms of dimension $[m]^2$. In the appendix the following expression for the quantity $I(l)$ is established

$$I(l) = \frac{1}{4\pi^2}\frac{v_1 \cdot v_2}{|l|^2}\left[1 + \alpha\sigma|l|^2 \alpha \ln\left(C\frac{\sigma}{\lambda^2}\right) + \mathcal{O}(\sigma^2|l|^4)\right], \quad (14)$$

where $\alpha \equiv \frac{3}{\pi}\frac{N_C}{N_C^2 - 1}(1 - \kappa)$ with the constant parameter estimated to be $\kappa \simeq 0.5$. As pointed out by Simonov [21], the first -and most important- term contributing to α comes from the paramegnetic, attractive interaction between the spin of the gluons with the non-perturbative background field, cf. Eq (5). It should also be noted that the constant C entering the argument of the logarithm is connected with the choice of parametrization of $D(z^2)$ see, eg., Ref [7]. In the context of a corresponding result having to do with an amplitude for a physically relevant, hence protected from infrared divergencies, process, then any such parameter would disappear.

Suppose the following problem is now posed: Given the above results, which pertain to the gluon propagation in a confining environment, look for an equivalent, effective particle-like mode propagation, summarizing their full content. In this spirit, we shall proceed to

assess the possibility that the gist of all we have done up to here can be reproduced via the introduction of an effective propagator. Following Ref [20] we make the substitution

$$\frac{1}{k^2} \to \frac{1}{k^2} + \frac{\mu^2}{k^4} \tag{15}$$

whose additional term signifies the presence of a linear term in the static potential. Then, one would obtain

$$I(l) = v_1 \cdot v_2 \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot l} \left(\frac{1}{k^2} + \frac{\mu^2}{k^4} \right). \tag{16}$$

The integral is infrared divergent and should require the introduction of a corresponding cutoff. Alternatively, one could restrict the validity of the replacement, according to Eq (15), to the region $k^2 > \mu^2$. Then, one would determine

$$\begin{aligned} I(l) &= v_1 \cdot v_2 \int\limits_{k^2>\mu^2} \frac{d^4k}{(\pi)^2} e^{ik\cdot l} \left(\frac{1}{k^2} + \frac{\mu^2}{k^4} \right) \\ &\simeq \frac{v_1 \cdot v_2}{|l|^2} \frac{1}{(2\pi)^4} \left[1 + \frac{\mu^2 |l|^2}{4} \ln \frac{4}{e\mu^2 |l|^2} + \mathcal{O}(\mu^4 |l|^4) \right] \quad \text{for}\, \mu^2|l|^2 < 1. \end{aligned} \tag{17}$$

Comparing the above result with that of Eq (14) one deduces

$$\mu^2 = 4\alpha\sigma = \frac{3}{\pi} \frac{N_c^2}{N_c^2 - 1} (1-\kappa)\sigma \simeq 2.15\sigma \simeq 0.4\,\mathrm{GeV}^2, \tag{18}$$

in full accord with the phenomenologically determined estimate for the tachyonic 'pole' [20-22]. One also observes that $\lambda \sim |l|\sigma$, as per our original supposition.

Returning to the original, full expression, which provides the full dynamical input for the amlitude, we write

$$\begin{aligned} E_{ii'jj'} &\simeq g^2 t^a_{ii'} t^a_{jj'} v_1 \cdot v_2 \int_{-\infty}^{+\infty} ds_2 \int_{-\infty}^{+\infty} ds_1 \frac{1}{4\pi^2 |l|^2} \left[1 + \sigma |l|^2 \alpha \ln \left(C \frac{\sigma}{\lambda^2} \right) \right] \\ &\simeq -g^2 t^a_{ii'} t^a_{jj'} v_1 \cdot v_2 \int_{-\infty}^{+\infty} ds_2 \int_{-\infty}^{+\infty} ds_1 \frac{1}{4\pi^2 |l|^2} \left[1 + \frac{\mu^2 |l|^2}{4} \ln \left(\frac{4}{e\mu^2 |l|^2} \right) \right] \\ &\simeq -g^2 t^a_{ii'} t^a_{jj'} v_1 \cdot v_2 \int_{-\infty}^{+\infty} ds_2 \int_{-\infty}^{+\infty} ds_1 \int\limits_{k^2>\mu^2} \frac{d^4k}{(2\pi)^2} e^{ik\cdot l} \left(\frac{1}{k^2} + \frac{\mu^2}{k^4} \right). \end{aligned} \tag{19}$$

Concerning the logarithmic factors entering the above result, it is useful to remark that the various constants appearing in the arguments are not of any particular importance -at least in the approximation we have been working- given that they would disappear with the appropriate choice for the infrared cutoff. More importantly, thinking in terms of the significance of the above results if they became part of an amplitude corresponding to a physically consistent, hence protected from infrared divergencies, process, then any dependence from these constants should be absent.

Going over to Minkowski space, the previous relation assumes the form

$$E_{ii'jj'} \simeq -g^2 t^a_{ii'} t^a_{jj'} i v_1 \cdot v_2 \int_{-\infty}^{+\infty} ds_2 \int_{-\infty}^{+\infty} ds_1 \int\limits_{k^2>\mu^2} \frac{d^4k}{(2\pi)^2} e^{-ik\cdot l} \left(-\frac{1}{k^2} + \frac{\mu^2}{k^4} \right). \tag{20}$$

Upon observing that

$$v_1 \cdot v_2 \int_{-\infty}^{+\infty} ds_2 \int_{-\infty}^{+\infty} ds_1 e^{-ik\cdot v_1 s_1 - ik\cdot v_2 s_2} = (2\pi)^2 v_1 \cdot v_2 \delta(k\cdot v_1)\delta(k\cdot v_2) = (2\pi)^2 \coth\gamma \delta(k_+)\delta(k_-), \tag{21}$$

where γ is determined by

$$\cosh\gamma \equiv \frac{v_1 \cdot v_2}{|v_1||v_2|} = \frac{s}{2m^2} \overset{s/m^2 \ll 1}{\Rightarrow} \gamma \simeq \ln(s/m^2) \Rightarrow \coth\gamma \simeq 1. \tag{22}$$

It follows

$$E_{ii'jj'} \simeq -\frac{g^2}{4\pi} t^a_{ii'} t^a_{jj'} i \coth\gamma\, f(b^2\mu^2) \tag{23}$$

where

$$f(b^2\mu^2) = \frac{1}{\pi} \int\limits_{k_\perp^2 > \mu^2} d^2k_\perp e^{ik_\perp \cdot b} \left(\frac{1}{k_\perp^2} + \frac{\mu^2}{k_\perp^4} \right). \tag{24}$$

One immediately notices that if $\mu^2 b^2 \ll 1$, then

$$f(\mu^2 b^2) \simeq \ln\left(\frac{4e}{\mu^2 b^2}\right), \tag{25}$$

which recovers the known perturbative result -with an infrared cutoff given by $\lambda^2 \equiv \frac{\mu^2}{4e}$.

As b grows, while remaining in the region $\mu^2 b^2 < 1$, one finds

$$f(\mu^2 b^2) \simeq \ln\left(\frac{4e}{\mu^2 b^2}\right)\left(1 - \frac{\mu^2 b^2}{4}\right) + \frac{1}{2}\mu^2 b^2. \tag{26}$$

In turn, this gives

$$E_{ii'jj'} \simeq -\frac{g^2}{4\pi} t^a_{ii'} t^a_{jj'} i \coth\gamma \left\{ \ln\left(\frac{4e}{\mu^2 b^2}\right)\left(1 - \frac{\mu^2 b^2}{4}\right) + \frac{1}{2}\mu^2 b^2 \right\}. \tag{27}$$

4. Summation of large logarithms

The presence of terms $\sim g^2 \ln 1/b^2\mu^2$, entering through the function $f(b^2\mu^2)$, imposes the need of their summation in the perturbative series. In the absence of the background field, i.e. in the framework of pQCD, it is well known that such a summation can be accomplished by employing the renormalization group strategies, which, for the quark-quark 'scattering' process under consideration, can be justified on the basis that b^{-1} plays the role of an ultraviolet cutoff. As Simonov has shown [11] the presence of the background field does not alter the, relevant for the summation, Callan-Symanzyk (C-S) equation. The physical basis on which this is so can be articulated by the following two arguments:

1. Contributions from the non-perturbative sector do not introduce additional divergencies, given that they are finite at short distances.

2. Dimension carrying quantities arising from the non-perturbative sector (correlators) are structured in terms of combinations of renormalization group invariant quantities gB, i.e. they behave as external momenta, as opposed to masses which are subject to renormalization.

Consequently, the called for renormalization group evolution follows the footsteps of the procedure employed in the purely perturbative analysis of the same 'process' [5]. In this connection it is recalled [2,26] that the Wilson contour configuration associated with $E_{ii'jj'}$ mixes with one corresponding to a pair of closed loops resulting from an alternative way of identifying the points at infinity [5]. It is associated with

$$\begin{aligned}\bar{E}_{ij'ji'} &= -\frac{\alpha_S}{\pi} c_F(\gamma \coth \gamma - 1)\delta_{ij'}\delta_{ji'} f(b^2\mu^2) \\ &\quad + \frac{\alpha_S}{\pi} c_F(\gamma \coth \gamma - 1 - i\pi \coth \gamma) t^a_{ij'} t^a_{ji'} i \coth \gamma f(b^2\mu^2) + \mathcal{O}(\alpha_S^2). \end{aligned} \tag{28}$$

Accordingly, and upon introducing, in shorthand notation, $W_1 \equiv \delta_{ii'}\delta_{jj'} + E_{ii'jj'}$ and $W_2 \equiv \delta_{ij'}\delta_{ji'} + \bar{E}_{ij'ji'}$, the C-S equation assumes the form

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} \right] W_a = -\Gamma_{ab} W_b, \;\; a, b = 1, 2, \tag{29}$$

with M playing the role of the uv cutoff whose running takes place between some lower scale (at which corresponding initial conditons are set) and an upper scale set by $1/b$. The anomalous dimension matrix Γ_{ab}, computed in the context of perturbation theory, see Refs [2,3,5], reads

$$(\Gamma_{ab}) = \frac{\alpha_s}{\pi} \left(\begin{array}{ll} -\frac{i\pi}{N} coth\gamma & i\pi coth\gamma \\ -\gamma coth\gamma + 1 + i\pi coth\gamma & N(coth\gamma - 1) - \frac{i\pi}{N} coth\gamma \end{array} \right) + \mathcal{O}(\alpha_S^2). \tag{30}$$

What *does* change, with respect to the perturbative analysis, on account of the the presence of non-perturbative, background contributions is the dependence of the W_a on the B-field correlators, i.e. one has $W_a = W_a[\{\Delta^{(n)}\}, M, g]$. Given that the computation has taken into account only the two-point correlator, the extra dependence of the W_a will involve the string tension, cf. Eq (13). The inclusion of this additional dimensional parameter will have its effects on the running coupling constant.

With this in mind, let us recast Eq (29) in integral form:

$$W_a[\sigma, M_2, g_B(M_2)] = \left\{ \mathrm{Pexp} \left[-\int_{M_1}^{M_2} \frac{dM}{M} \Gamma(g_B(M)) \right] \right\}_{ab} W_b[\sigma, M_1, g_B(M_1)] \tag{31}$$

with the path ordering becoming necessary because the anomalous dimension matrices do not commute with each other. Concerning the integration limits a consistent choice, given the premises of the present calculation, is to take $M_2 = 1/b$ and set $M_1 = 1/b_0$ with $b_0^2\sigma < 1$. It is observed that the non-perturbative input enters the W_a not only through their explicit dependence on the string constant, but also -which is the most impotant- through a running coupling constant g_B, which obeys the equation

$$M\frac{\partial}{\partial M} g_B(M) = \beta(g_B(M)). \tag{32}$$

The solution of the latter calls for initial conditions which are influenced by the presence of the non-perturbative background and specifically by σ. Such matters have been studied by Simonov in [25].

Turning our attention to the 'deformation' (to the one-loop order) of the running coupling constant, on account of its additional dependence on the background field, we proceed as follows. Knowing the perturbative result to order α_S, we go to Eq (31) and present its solution in the form

$$W_1[\sigma, 1/b, \alpha_B(1/b)] = \delta_{ii'}\delta_{jj'} - i\coth\gamma \left[\alpha_S(1/b_0^2) f(b_0^2\mu^2) \int_{1/b_0^2}^{1/b^2} \frac{d\tau}{\tau}\alpha_S(\tau)\right] t^a_{ii'} t^a_{jj'} + \mathcal{O}(\alpha_S^2). \quad (33)$$

It follows that

$$\int_{1/b_0^2}^{1/b^2} \frac{d\tau}{\tau}\alpha_B(\tau) = \alpha_S(1/b^2) f(b^2\mu^2) - \alpha_S(1/b_0^2) f(b_0^2\mu^2) + \mathcal{O}(\alpha_S^2). \quad (34)$$

Upon comparing with Eq (27) one obtains

$$\alpha_B(\tau) = \alpha_S(\tau)\left\{1 + \frac{\mu^2}{4\tau}\left[\ln\left(\frac{4\tau}{\mu^2}\right) - 2\right]\right\} + \mathcal{O}(\alpha_S^2). \quad (35)$$

It should be noted that the validity of the above results holds for $\tau/\mu^2 > 1$ and $\alpha_S(\tau) = \frac{4\pi}{\beta_0}\frac{1}{\ln(\tau/\Lambda^2)} < 1$. An indicative estimate, on the basis of Eq (35), is that if $\alpha_S \simeq 0.5$, then $\alpha_B \simeq 0.5(1+0.05)$. Following Refs [3,5], one surmises that the 'amplitude' A for the 'process' under consideration behaves as

$$A \sim \exp\left[-\frac{N_C}{2\pi}\ln\left(\frac{s}{m^2}\right)\int_{1/b_0^2}^{1/b^2}\frac{d\tau}{\tau}\alpha_B(\tau) + \mathcal{O}(\alpha_S^2)\right] \propto \exp\left[-\frac{\alpha_S}{2\pi}N_C\ln\left(\frac{s}{m^2}\right) f(b^2\mu^2)\right]. \quad (36)$$

from which one 'reads' a reggeized behavior for the gluon. The difference from the usual, purely perturbative result is that the function $f(b^2\mu^2)$ is now connected with the modified propagator, as per Eq (24).

In conclusion, we have demostrated that the non-perturbative input, through the SVM, to the analysis of a hypothetical quark-quark 'scattering' process in the Regge kinematical region, produces a result which, in a phenomenological context, has been argued to be extremely attractive in reproducing low energy hadron phenomenology. In a sense, this investigation could be considered as a special example, which justifies Simonov's more general argumentation [22] according to which the perturbative-nonperturbative interference in static QCD interactions at small distances imply the presence of a linear term in the potential.

Appendix

Given the set of the defining, worldline formulas given by Eqs (6)-(15) in section 3, we proceed to derive derive Eq (16). In the course of the derivation the various quantities and parameters appearing in the last part of the section are specified.

Writing

$$D(z^2) = \int_0^\infty dp\tilde{D}(p)e^{-pz^2} \tag{A.1}$$

one determines

$$\begin{aligned} I(l) &= \frac{1}{4\pi^2}\frac{v_1\cdot v_2}{|l|^2}\frac{2N_c^2}{Nc^2-1}v_1\cdot v_2\int_0^1 d\alpha_2\alpha_2\int_0^1 d\alpha_1\alpha_1\int_0^1 dt_2\int_0^1 dt_1\,\theta(t_2-t_1) \\ &\times\int_0^\infty dp\,\tilde{D}(p)\,[48Q(p;t_2,t_1)-R(p;t_2,t_1,\alpha_2\alpha_1)]+\mathcal{O}(<g^4F^4>_B), \end{aligned} \tag{A.2}$$

where we have made the change $t_i \to Tt_i$, $i = 1, 2$ and have introduced the quantities

$$Q(p;t_2,t_1) \equiv \left(\frac{\pi}{p}\right)^2\int_0^\infty dT\,e^{-T\lambda^2}\int\frac{d^4q}{(2\pi)^4}e^{-\frac{q^2}{4p}}\int\limits_{\substack{x(0)=0\\x(1)=l}}\mathcal{D}x(t)e^{-\frac{1}{4T}\int_0^1 dt\dot{x}^2(t)}e^{iq\cdot(x(t_2)-x(t_1))} \tag{A.3}$$

and

$$\begin{aligned} R(p;t_2,t_1,\alpha_2,\alpha_1) &\equiv \left(\frac{\pi}{p}\right)^2\int_0^\infty dT\,e^{-T\lambda^2}\int\frac{d^4q}{(2\pi)^4}e^{-\frac{q^2}{4p}}(\delta_{\mu_2\nu_2}\delta_{\mu_1\nu_1}-\delta_{\mu_2\nu_1}\delta_{\mu_1}) \\ \times\int\limits_{\substack{x(0)=0\\x(T)=l}}&\mathcal{D}x(t)e^{-\frac{1}{4}\int_0^T dt\dot{x}^2(t)}\dot{x}_{\mu_2}(t_2)x_{\nu_2}(t_2)\dot{x}_{\mu_1}(t_1)x_{\nu_1}(t_1)e^{[iq\cdot(\alpha_2x(t_2)-\alpha_1(t_1))]} \end{aligned} \tag{A.4}$$

with $|l|$, as defined in the text.

The above path integrals can be executed by employing standard techniques, given that 'particle' action functionals are quadradic (plus a linear term) [16-18]. Ignoring terms giving contributions $\mathcal{O}(b^2)$ and using condensed notation from hereon, one determines

$$Q = \frac{1}{16}\frac{1}{p^2}\int_0^\infty dT\,e^{-T\lambda^2}\int\frac{d^4q}{(2\pi)^4}e^{-\frac{q^2}{4p}-Tq^2G_{12}}[1+\mathcal{O}(l^2q^2)] \tag{A.5}$$

and

$$R = \frac{1}{16}\frac{1}{p^2}\int_0^\infty dT\,e^{-T\lambda^2}\int\frac{d^4q}{(2\pi)^4}e^{-\frac{q^2}{4p}-Tq^2K_{12}}[c_0+Tq^2c_1+T^2q^4c_2++\mathcal{O}(l^2q^2)], \tag{A.6}$$

where the following, one dimensional particle propagator-type quantities have been introduced

$$\Delta_{12} = \Delta(t_1,t_2) \equiv t_1(1-t_2)\theta(t_2-t_1)+t_2(1-t_1)\theta(t_1-t_2), \tag{A.7}$$

$$G_{12} = G(t_2,t_1) = \Delta(t_2,t_2)+\Delta(t_1,t_1)-2\Delta(t_1,t_2) = |t_2-t_1|(1-|t_2-t_1|) \tag{A.8}$$

and

$$K_{12} = K(t_2, t_1) = \alpha_2^2 \Delta(t_2, t_2) + \alpha_1^2 \Delta(t_1, t_1) - 2\alpha_1\alpha_2\Delta(t_1, t_2). \tag{A.9}$$

The coefficients entering Eq(A.6) are given by the expressions

$$c_0 = -72\Delta_{12}\partial_1\Delta_{12}\partial_2\Delta_{12} \tag{A.10}$$

$$\begin{aligned} c_1 \;=\; & 48\Delta_{12}\partial_2\Delta_{12}\partial_1 K_{12} + 24\alpha_1\partial_1\Delta_{12}\partial_2\Delta_{12}(\alpha_1\Delta_{11} - \alpha_2\Delta_{12}) \\ & +24\alpha_2\partial_1\Delta_{12}\partial_2\Delta_{12}(\alpha_2\Delta_{22} - \alpha_1\Delta_{12}) - 12(\alpha_1^2\Delta_{12}\partial_1\Delta_{11}\partial_2\Delta_{12} + \alpha_2^2\Delta_{12}\partial_2\Delta_{22}\partial_1\Delta_{12} \\ & -\alpha_1\alpha_2\Delta_{12}\partial_1\Delta_{11}\partial_2\Delta_{22} - \alpha_1\alpha_2\Delta_{12}\partial_1\Delta_{12}\partial_2\Delta_{12}) \\ & -12(\alpha_2\Delta_{22} - \alpha_1\Delta_{12})(\alpha_1\partial_1\Delta_{11}\partial_2\Delta_{12} - \alpha_2\partial_1\Delta_{12}\partial_2\Delta_{12}) \\ & -12(\alpha_1\Delta_{11} - \alpha_2\Delta_{12})(\alpha_2\partial_2\Delta_{22}\partial_1\Delta_{12} - \alpha_1\partial_1\Delta_{12}\partial_2\Delta_{12}) \end{aligned} \tag{A.11}$$

and

$$c_2 = 24(\alpha_2\Delta_{22} - \alpha_1\Delta_{12})(\alpha_1\Delta_{11} - \alpha_2\Delta_{12})\partial_1 K_{12}\partial_2\Delta_{12}. \tag{A.12}$$

Given the above, the "paramagnetic" contribution to Eq (A.2) becomes

$$\begin{aligned} I_p \;&=\; 12\frac{2N_c}{Nc^2 - 1} v_1 \cdot v_2 \int_0^\infty dp\, \tilde{D}(p) \int_0^1 dt_2 \int_0^1 dt_1\, \theta(t_2 - t_1) Q(p; t_2, t_1) \\ &= \frac{12}{16}\frac{2N_c^2}{N_c - 1}\frac{v_1 \cdot v_2}{4\pi^2}\int_0^\infty \frac{dp}{p}\tilde{D}(p)\left[\ln\left(4e^{-\gamma_E}\frac{p}{\lambda^2}\right) + \mathcal{O}(\lambda^2/p)\right] \end{aligned} \tag{A.13}$$

and since

$$\int_0^\infty \frac{dp}{p}\tilde{D}(p) = \int_0^\infty dz^2\, \tilde{D}(z^2) = \frac{1}{\pi}\int_0^\infty d^2z\, \tilde{D}(z^2) \equiv \frac{2}{\pi}\sigma, \tag{A.14}$$

Eq (A.9) gives

$$I_p = \frac{3N_c}{N_c^2 - 1}\frac{v_1 \cdot v_2}{4\pi^2}\sigma \ln\left(C\frac{\sigma}{\lambda^2}\right). \tag{A.15}$$

This furnishes the correction term from the background gauge field contributions entering Eq (14) in the text. The constant C entering the above result depends on the parametrization of $D(z^2)$. Following the one of Nachtman [7], one determines $C = 39.65$. The numerical computation of the factor κ, based on the expressions for the c_i, as given by (A.11), produces the value $\kappa \simeq 0.5$.

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